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# Memory integer model parameters based on polyurethane foam experimental compression tests

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**ABSTRACT.** *The polyurethane foam (PU) undergoing large compressive deformation exhibits highly nonlinear elasticity and viscoelastic behaviour. The principal aim of this paper is to modeling this behaviour using a memory integer model which describes the nonlinearity by polynomial function and the viscoelasticity by convolution function.*

*A unidirectional compression tests are considered to identify experimentally the mechanical parameters of model. The difference between force responses of foam in load and unload phases constitute the base element of the identification methodology. Many precautions are taken into account to find accurate results that verify thermodynamics conditions. Finally, the reliability and also the limits of the memory integer model are discussed.*

**RÉSUMÉ.** *Les mousses de polyuréthane sous l'effet de fort taux de compression présentent un comportement à la fois fortement élastique non linéaire, et viscoélastique. L'objectif de cet article est de modéliser ce comportement en utilisant un modèle à mémoire entier qui modélise l'élasticité non linéaire par une fonction polynomiale, et l' viscoélasticité par une fonction de convolution.*

*A partir d'essais de compression uni-axial, nous avons cherché à déterminer les paramètres du modèle. La méthode de différence entre la charge et la décharge constitue la base de la méthode d'identification. De nombreuses précautions ont été prises pour obtenir des résultats fiables et précis, ainsi que pour vérifier les conditions thermodynamiques de stabilité du modèle. Enfin, la validité et les limites du modèle à mémoire entier sont discutées.*

**KEY WORDS:** *Flexible polyurethane foam, quasi-static behaviour, linear viscoelasticity, thermodynamics conditions of model, memory integer model, identification parameters.*

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*MOTS-CLÉS: Mousse de polyuréthane, comportement quasi-statique, viscoélasticité linéaire, conditions thermodynamiques, modèle à mémoire entier, identification des paramètres.*

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## 1. Introduction

Foam is defined as a solid phase arranged as polyhedral (called cells) which packs in the three dimensions to fill the space. If the faces connecting the cells are opened, the foam is said to be open-celled. Such materials are common in nature, we call them wood, cork or cancellous bone. Foams and all the cellular materials with low densities have nowadays more and more importance in many fields of industry. They are used, for example as a construction material for thermal or sound insulation and for the absorption of large vibrations. They are also widely used for the comfort of automobile seats.

The literature on foam is a large one, composed as well as published paper than books. The foam has been studied in static conditions (Gibson *et al.*, 1997, Tu *et al.*, 2001, Goangseup *et al.*, 2008), quasi-static conditions (Deng *et al.*, 2006, Ouellet *et al.*, 2006, Dupuis *et al.*, 2008, Ippili *et al.*, 2008, Njeugna *et al.*, 2008), or dynamic conditions (White, 1998, Singh *et al.*, 2001, Singh *et al.*, 2003, Singh *et al.*, 2003(2), Deng, 2004, Ippili *et al.*, 2008). The long term behaviour (Bezazi *et al.*, 2009, Rizov *et al.*, 2009) to ensure the durability of mechanical properties was also explored. The reference in static and quasi-static is undoubtedly the work of Gibson and Ashby (Gibson *et al.*, 1997). Thus, they have shown that the characteristics of foams depend on various parameters such as cell morphology, size, arrangement, connections between them... Static mechanical behaviour of foams has been well described by Gibson and Ashby (Gibson *et al.*, 1997), particularly in the effect of a compressive force. On the stress- strain curves of polyurethane foam, three main parts can be distinguished. First a linear elasticity at low stress followed by a long collapse plateau, and finished by a densification of the material in which the stress rises steeply. The hysteresis cycle in compression highlights the viscoelastic behaviour of polyurethane foam, especially under large level of compressive deformations.

To predict the mechanical response of polyurethane foams, two analysis modes are distinguished namely the microscopic analysis and the macroscopic analysis. The micro-mechanical models consist in considering the fundamental components of foam such as shape cells, facets and beams cell. The mechanical response is predicted by using finite elements method or finite volumes method or also Voronoi technique that supposes a random distribution of different shape cell (Zhu *et al.*, 2006, Song *et al.*, 2010). This analysis develops relationships between the geometric properties of cells and the elastic properties of foam. It shows that foam undergoing large deformation exhibits a nonlinear elastic behaviour. However, this analysis is not able to predict dynamics properties. The macroscopic analysis considers the overall response of a foam sample in order to estimate the macro-mechanical material parameters such as quasi-static and dynamic stiffness, damping coefficient and viscoelastic compounds. In the framework for this analysis, there are two types of models: energetic models (Dupuis *et al.*, 2008) and memory models (integer memory model (Singh *et al.*, 2001, Singh *et al.*, 2003, Singh *et al.*, 2003(2), Ippili *et al.*, 2008) and the fractional memory model (Deng, 2004, Deng *et al.*, 2006)). The

memory models illustrate the historical effect on the behaviour of the foam. They describe the nonlinearity of elastic behaviour by a polynomial function and the viscoelasticity by a convolution function.

To identify the integer model parameters, Singh R. consider a dynamic compression test. In his first work, he models polyurethane foam by a linear integer model (linear elastic behaviour with convolution function for viscoelastic behaviour) and develops an analytical identification process in the case of low amplitude impact test (Singh *et al.*, 2001). In his second work, Singh R. consider also a linear integer model and estimate the dynamic properties of foam through use of Prony series (Singh *et al.*, 2003). In a final work, Singh R. identifies the nonlinear integer model parameters by using the balance harmonic when the input test (displacement) is sinusoidal (Singh *et al.*, 2003). Recently, Ippili R.K. considers a quasi-static compression test (Ippili *et al.*, 2008). In his work, the average curve between load and unload phases constitute the initial elastic polynomial for his method. It will be then adjusted in order to obtain the best correspondence between experimental and model force-displacement curves. The viscoelastic parameters are obtained by using a linear regression estimation method (e.g. ARMA method). This method will called average force method. To identify the fractional model parameters, Deng R. develops a method based on the elastic force symmetry (Deng *et al.*, 2006). The experimental curve, representing the difference forces between the load and unload phases, is then determined and the viscoelastic parameters are identified by using minimizing methods of average least square error between experimental difference force between load and unload and the same model curve.

In this paper, the method of difference forces between load and unload phases, considered only by Deng R. for his fractional model, has been adapted for the integer memory model to understand the quasi-static behaviour of polyurethane foam. To identify the viscoelastic model parameters, an optimization approach in three steps has been conceived. This approach allows finding the best parameter combination that verifies the causality and stability conditions of the model. The results obtained in this work differ from the results obtained by Ippili R.K. considering the integer model and the average force method (Ippili *et al.*, 2008). The limit of the integer model has been deduced and the results have been discussed.

## 2. Experimental study

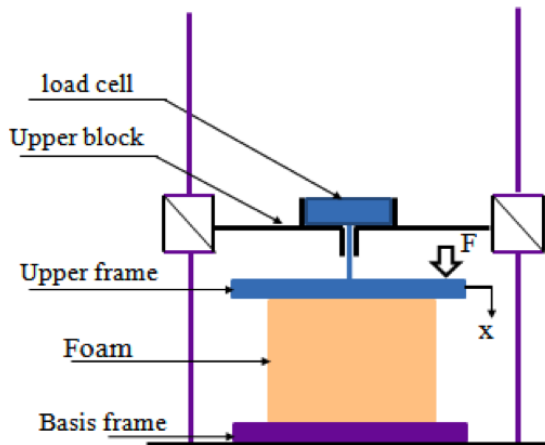
### 2.1. Polyurethane foam samples

Our samples of polyurethane foam called Foam Type A have been used in all the tests. To ensure reliable results, the all samples used have the same mechanical and environmental history. This type of foam has been chosen because its characteristics are similar as seat cars foam (Table 1).

Material	Characteristics
Foam type	Flexible polyurethane foam
Designation	Foam Type A
Relative Density	$28 \text{ kg m}^{-3}$
Porosity	$820 \mu\text{m}$
Dimensions ( $L_0 \times l_0 \times h_0$ )	$0.75 \text{ m} \times 0.75 \text{ m} \times 0.75 \text{ m}$
Cells type	open

**Table 1.** Chemical and morphological characteristics of Foam Type A

## 2.2. Quasi-static compression test device and conditions



**Figure 1.** Quasi-static compression test device.

The quasi-static compression device ‘Instron 33R4240’ (tension-compression machine) is composed by a non-moving basis frame and upper block which moves vertically. This machine is driven by Bluehill 2 software which allows the definition of the test conditions (maximum compression level, strain rate, etc.), the sampling period  $T_{ech}$  and mechanical properties to extract. The quasi-static test consists in compressing the foam sample between the upper frame and the basis machine until

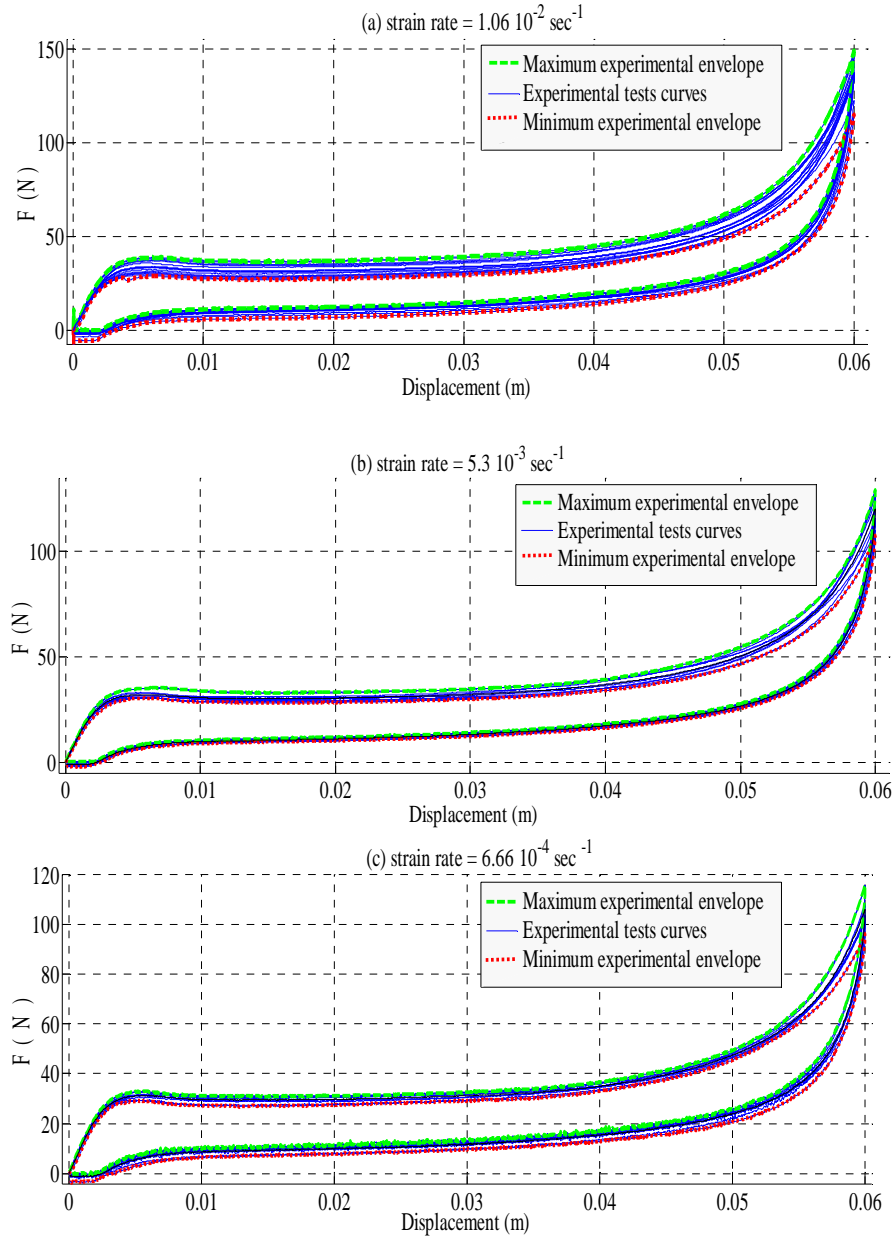
final compression level (load phase) and in unloading progressively until initial compression level (unload phase, second step). To minimize the noise contribution, the maximum experimental response force of foam must be slightly less than the load cell maximum capacity. Displacement and force sensors are already integrated in the machine.

To ensure reliable and generalizable results, 81 specimens of foam Type A have been used (Table 2:  $N=45$  for the test n°1,  $N=17$  for the test n°2, and  $N=19$  for the test n°3). All specimens have the same mechanical and environmental histories: they are virgin specimens and have been obtained by cutting mattresses of dimension 2000 mm x 1200 mm x 75 mm into cubic samples. Each specimen has been compressed only one time. The test conditions are summarized in the table 2.

	$\dot{\epsilon}$ (sec <sup>-1</sup> )	$\epsilon_0$ (%)	$\epsilon_{\max}$ (%)	$T$ (sec)	$T_{ech}$ (sec)	$N$
Test n°1	$1.06 \cdot 10^{-2}$	0	80	150	0.0625	45
Test n°2	$5.30 \cdot 10^{-3}$	0	80	300	0.125	17
Test n°3	$6.66 \cdot 10^{-4}$	0	80	2400	2	19

**Table 2.** *Quasi-static compression test conditions.*

The experimental force-displacement curves of foam Type A, are presented in Figure 2. The tests allow extracting the minimum and maximum experimental envelopes. The experimental results of tests exhibit a hysteresis cycle proving the highly viscoelastic comportment of foam. It is also observed that the dispersion during load seems more important than the dispersion during unload phase. All foam samples return almost by the same unload way.



**Figure 2.** Experimental force-displacement curves (--- Maximum experimental envelope, ··· Minimum experimental envelope, — Experimental test curves).

### 3. Modelling study

The polyurethane foam undergoing large compressive deformation is assumed to be homogeneous, isotropic with constant cross-section. These hypotheses have been considered by all researchers who have been worked on macroscopic memory models. In this case, the strain and the stress are proportional respectively to displacement and to force. So, it is possible to consider the force-displacement response to describe the quasi-static behaviour of polyurethane foam. We choose this description for eventual comparison with the results of Deng R. (Deng *et al.*, 2006) and Ippili R.k. (Ippili *et al.*, 2008) in further work.

The polyurethane foam undergoing large compressive deformation exhibits highly nonlinear elasticity and viscoelastic behaviour.

#### 3.1. Nonlinear integer model

The global model of foam is composed by an elastic part and a viscoelastic part:

$$F(t) = F_e(t) + F_{ve}(t) \quad [1]$$

The elastic foam component is typically modelled by a nonlinear spring described by a polynomial function:

$$F_e(t) = \sum_{i=1}^M K_i (x(t))^i \quad [2]$$

In literature many elementary macroscopic models are used to describe the viscoelastic behaviour such as Maxwell model, Kelvin Voigt model and Zener model. Assuming a random combination of elementary models in series and parallel the viscoelastic behaviour is given by:

$$b_0 F_{ve}(t) + b_1 \frac{d^1 F_{ve}(t)}{dt} + \dots + b_r \frac{d^r F_{ve}(t)}{dt^r} = c_0 x(t) + c_1 \frac{d^1 x(t)}{dt} + \dots + c_n \frac{d^n x(t)}{dt^n} \quad [3]$$

The integer model of viscoelasticity [4] is obtained when  $r > n$ ,  $r > 2$  and all the poles of the impulsion response are distinct. In this work P is considered equal two.

$$F_{ve}(t) = \int_0^t \sum_{l=1}^P a_l e^{-\alpha_l(t-\tau)} x(\tau) d\tau \quad [4]$$



If  $r=n$ ,  $r>2$  and  $c_0=0$ , the viscoelastic force is generally composed by purely elastic component and memory term:

$$F_{ve}(t) = K_{ve}x(t) + \int_0^t \sum_{l=1}^P a_l e^{-\alpha_l(t-\tau)} \dot{x}(\tau) d\tau \quad [5]$$

The global model of foam undergoing large compression deformation is given by:

$$F(t) = F_{ve}(t) + F_e(t) = \int_0^t \sum_{l=1}^P a_l e^{-\alpha_l(t-\tau)} \dot{x}(\tau) d\tau + \sum_{i=1}^N k_i (x(t))^{\gamma_i} \quad [6]$$

### 3.2. Thermodynamic conditions

The thermodynamic conditions are the causality and stability conditions of model. They are determinate from the Fourier transform  $G(j\omega)$  of impulse model response neglecting nonlinear components.

$$G(j\omega) = k_1 + \sum_{l=1}^P \frac{a_l}{j\omega - \alpha_l} \quad [7]$$

To ensure the stability and the causality of model, it's necessary to verify the following conditions:

$$\begin{cases} \operatorname{Re}(G(j\omega)) \geq 0 & \forall \omega \geq 0 \\ \operatorname{Im}(G(j\omega)) \geq 0 & \forall \omega \geq 0 \end{cases} \quad [8]$$

The thermodynamic conditions of integer model are grouped in the table 3.

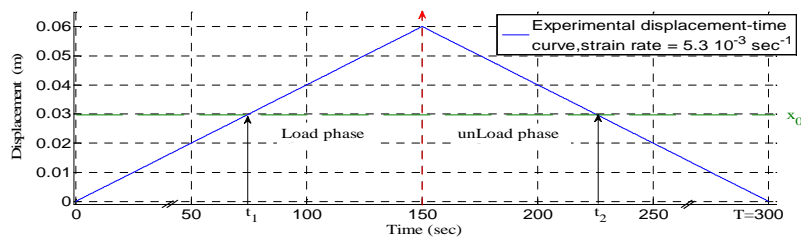
The viscoelastic parameters vector to determinate: $X = [\text{Re}(\alpha_1), \text{Im}(\alpha_1), \text{Re}(a_1), \text{Im}(a_1)]_{p=2}$	
$\omega \rightarrow +\infty$	$\omega \rightarrow 0$
$k_1 \geq 0$ $X(3) \leq 0$	$\left\{ \begin{aligned} &k_1 \left( (X(1)^2 + X(2)^2) + 4X(1)^4 \right) \\ &+ (X(2)X(4) + X(1)X(3))(X(1)^2 + X(2)^2) \end{aligned} \right\} \geq 0$ $X(2)X(4) + X(1)X(3) \leq 0$
$X(1) \geq 0$ (stability condition of viscoelastic component)	

**Table 3.** Thermodynamic conditions for integer model.

#### 4. Identification method

##### 4.1. Difference-forces method

In the case of quasi-static compression test, the strain rates in load and unload are considered equal. So, the displacement curve  $x(t)$  is presented by isosceles triangle (Figure 3).

**Figure 3.** Experimental displacement-time curve.

The times  $(t_1, t_2)$  corresponding to the same displacement  $x_0$  respectively in loading and unloading phases are related by:

$$t_2 = T - t_1 \quad [9]$$

The approach of the difference-forces method is based on the symmetry of the elastic force between load and unload phases. So only the viscoelastic parameters figure in the analytical expression of difference forces between load and unload:

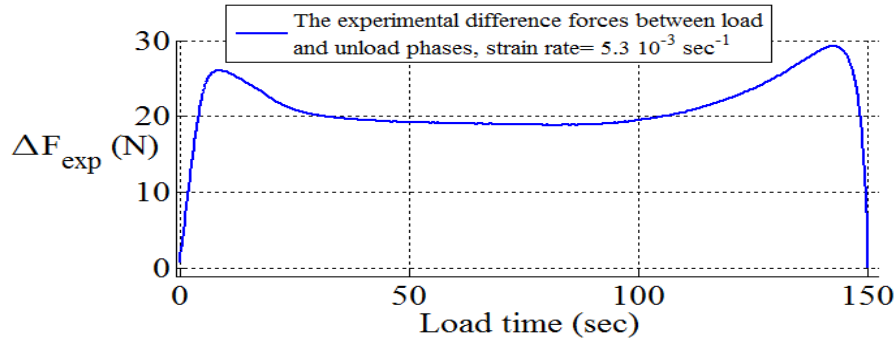
$$\Delta F_{th}(x_0) = \int_0^{t_1} \sum_{i=1}^P a_i e^{-\alpha_i(t_1-\tau)} x(\tau) d\tau - \int_0^{t_2} \sum_{i=1}^P a_i e^{-\alpha_i(t_2-\tau)} x(\tau) d\tau \quad [10]$$

Using the previous equations [9] and [10], the load time projection of difference forces is determined:

$$\Delta F_{th}(t_1) = \sum_{i=1}^P \frac{l_0 \dot{\epsilon}}{\alpha_i^2} a_i \left( e^{-\alpha_i t_1} + 2e^{\alpha_i \left( t_1 - \frac{T}{2} \right)} - e^{-\alpha_i (T-t_1)} - 2 \right) \quad [11]$$

The difference-forces method consists in determining the viscoelastic parameters. We use an optimization method, which allows minimizing the least mean square error between analytical and experimental difference forces (Figure 4). In the second step, after reconstructing the viscoelastic force in load and unload phases, the determination of the elastic force becomes possible by simple subtraction. Then, the symmetry condition of elastic force must be verified and the elastic polynomial coefficients can easily identified using MATLAB.

Note that, in this work P has been considered equal two because the results obtained with three and four don't change significantly.



**Figure 4.** The projection on load time of difference of forces between load and unload phases.

#### 4.2. Optimization approach

Optimization methods are the basic tools for viscoelastic parameters identification. There are two types of optimization methods namely deterministic methods and random methods. Deterministic methods are effective when the objective function has a known prior form so that it is possible to choose an initialization near the global minimum. However, in the general case, these methods are not able to find the global minimum. The random methods are reliable in the case of non-differentiable functions and noisy functions. In addition, these methods offer a great probability to find the global minimum when the objective function possesses many local minima. However, his calculation time is very important comparing with deterministic methods. In order to exploit the advantages of deterministic methods while ensuring a high probability to find global minimum, an approach involves three steps is considered. It consists in using both the genetic algorithm ('gatool' command of MATLAB) and the Trust region reflective method ('lsqnonlin' command). The first step allows having a great chance to find initialization for 'lsqnonlin MATLAB solver' near global optimum by using the genetic algorithm with gross stopping criteria. In the second step, we search the best combination of viscoelastic parameters which minimizes the least mean square error between analytical and experimental difference forces. This combination is finalized by using 'lsqnonlin MATLAB solver' with tight stopping criteria. Finally, the solution found in the second step is injected into random initial population of genetic

algorithm to verify that it is effectively the global minimum. The stopping criterion in this third step is the generation number fixed at 100.

## 5. Results and discussion

### 5.1. Results

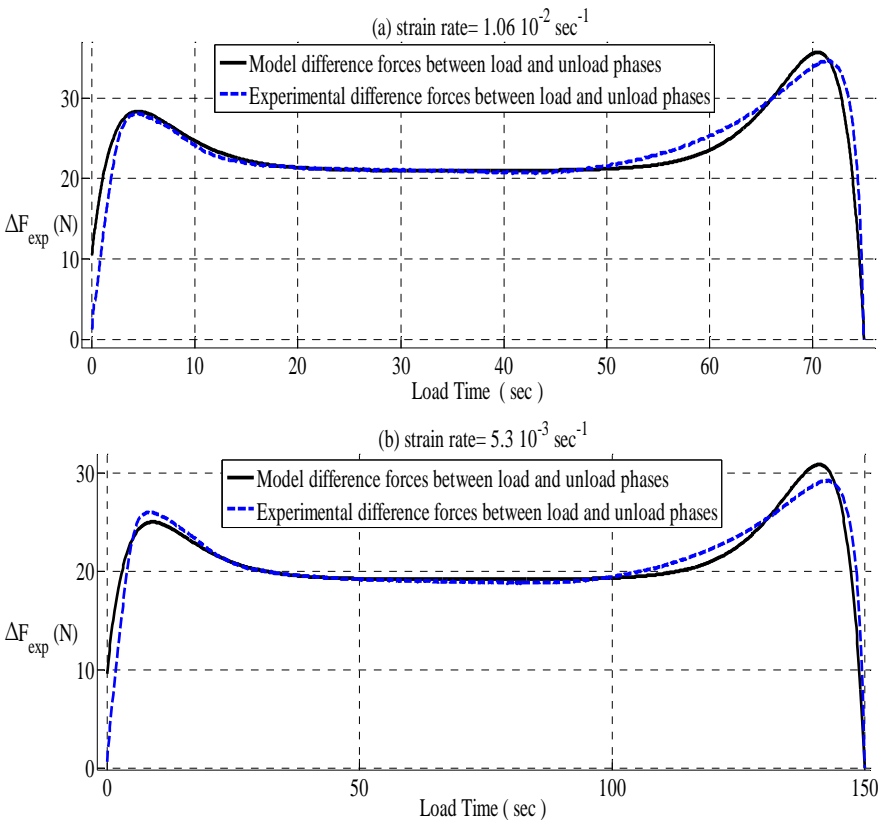
The identification results are regrouped in the following table:

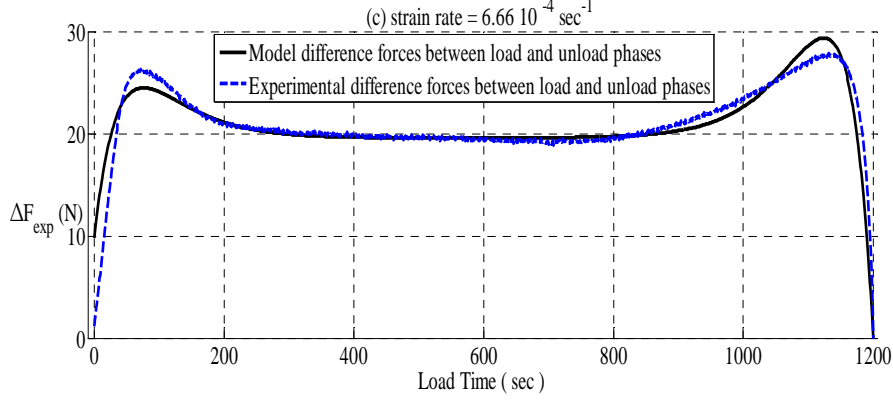
Parameters	Test n°1 ( $\dot{\epsilon} = 1.06 \cdot 10^{-2}$ sec <sup>-1</sup> )	Test n°2 ( $\dot{\epsilon} = 5.30 \cdot 10^{-3}$ sec <sup>-1</sup> )	Test n°3 ( $\dot{\epsilon} = 6.66 \cdot 10^{-4}$ sec <sup>-1</sup> )
$X(1)$ (sec <sup>-1</sup> )	0.302	0.155	$1.925 \cdot 10^{-4}$
$X(2)$ (sec <sup>-1</sup> )	$1.49 \cdot 10^{-3}$	$7.58 \cdot 10^{-4}$	$9.31 \cdot 10^{-5}$
$X(3)$ (N m <sup>-1</sup> sec <sup>-1</sup> )	-3906.46	-1719.87	-192.93
$X(4)$ (N m <sup>-1</sup> sec <sup>-1</sup> )	332351.75	146322.49	16187.15
$\Delta F$ error (%)	5.07	5.63	4.73
$M$	8	8	8
$k_1$ (N m <sup>-1</sup> )	$2.01 \cdot 10^4$	$1.88 \cdot 10^4$	$1.78 \cdot 10^4$
$k_2$ (N m <sup>-2</sup> )	$-4.32 \cdot 10^5$	$-6.27 \cdot 10^5$	$-7.28 \cdot 10^5$
$k_3$ (N m <sup>-3</sup> )	$9.4 \cdot 10^6$	$31.4 \cdot 10^6$	$44.4 \cdot 10^6$
$k_4$ (N m <sup>-4</sup> )	$7.50 \cdot 10^8$	$-5.21 \cdot 10^8$	$-13.38 \cdot 10^8$
$k_5$ (N m <sup>-5</sup> )	$-5.570 \cdot 10^{10}$	$-1.380 \cdot 10^{10}$	$1.478 \cdot 10^{10}$
$k_6$ (N m <sup>-6</sup> )	$1.534 \cdot 10^{12}$	$7.410 \cdot 10^{11}$	$1.675 \cdot 10^{11}$
$k_7$ (N m <sup>-7</sup> )	$-1.972 \cdot 10^{13}$	$-1.170 \cdot 10^{13}$	$-5.540 \cdot 10^{12}$
$k_8$ (N m <sup>-8</sup> )	$9.89 \cdot 10^{13}$	$6.50 \cdot 10^{13}$	$3.76 \cdot 10^{13}$
Symmetry error (%)	0.980	2.470	0.057
Elasticity identification error (%)	0.070	0.045	0.063

**Table 4.** Parameters identification results.

5.2. Validation of results

In order to validate these results, we decided to examine five criteria namely thermodynamic conditions, statistic quality, good  $\Delta F$  estimation, model force response between experimental envelopes and elastic symmetry hypothesis.





**Figure 5.** Comparison between model and experimental difference-forces.

The parameters results check the thermodynamic conditions presented in table 3.

They are the mean of  $N$  test samples (Table 2.) which is determined to ensure the statistical quality of all identified parameters for each strain rate (second criteria). This quality is reviewed through the set at a 95% confidence level and the statistical error limit "SLE" which must not exceed 10%. The minimum number of test samples for each strain rate is calculated using the following equation:

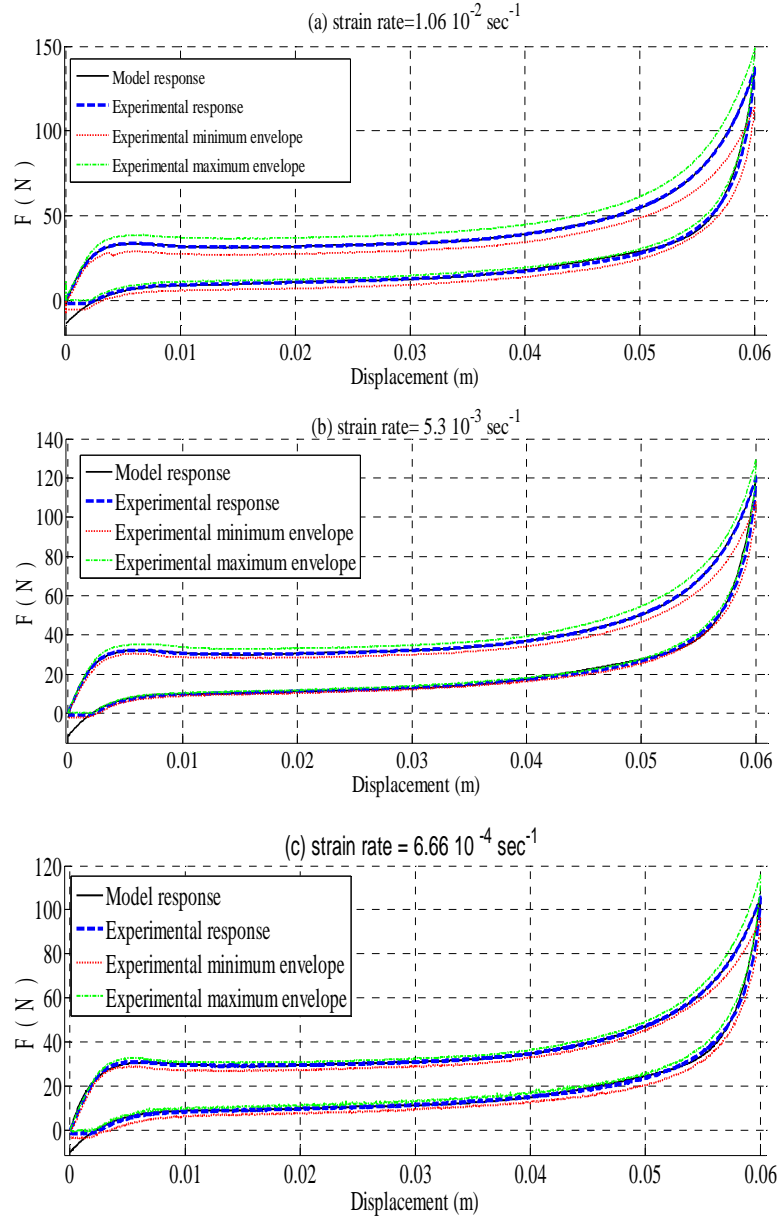
$$N = \max_l \left( 1 + \text{ceil} \left( \left( \frac{100}{SLE} \frac{u_l \hat{\sigma}_l}{\hat{m}_l} \right)^2 \right) \right) \quad [12]$$

$(\hat{m}_l, \hat{\sigma}_l)$  are the estimated average and standard deviation values corresponding to  $l^{th}$  parameter. They are calculated from the identification results of the preliminary test battery (15 test samples).  $u_l$  is a coefficient determined from a probability table suitable for the estimated probability law of  $l^{th}$  parameter. In this paper, the Student law is assumed for all parameters.

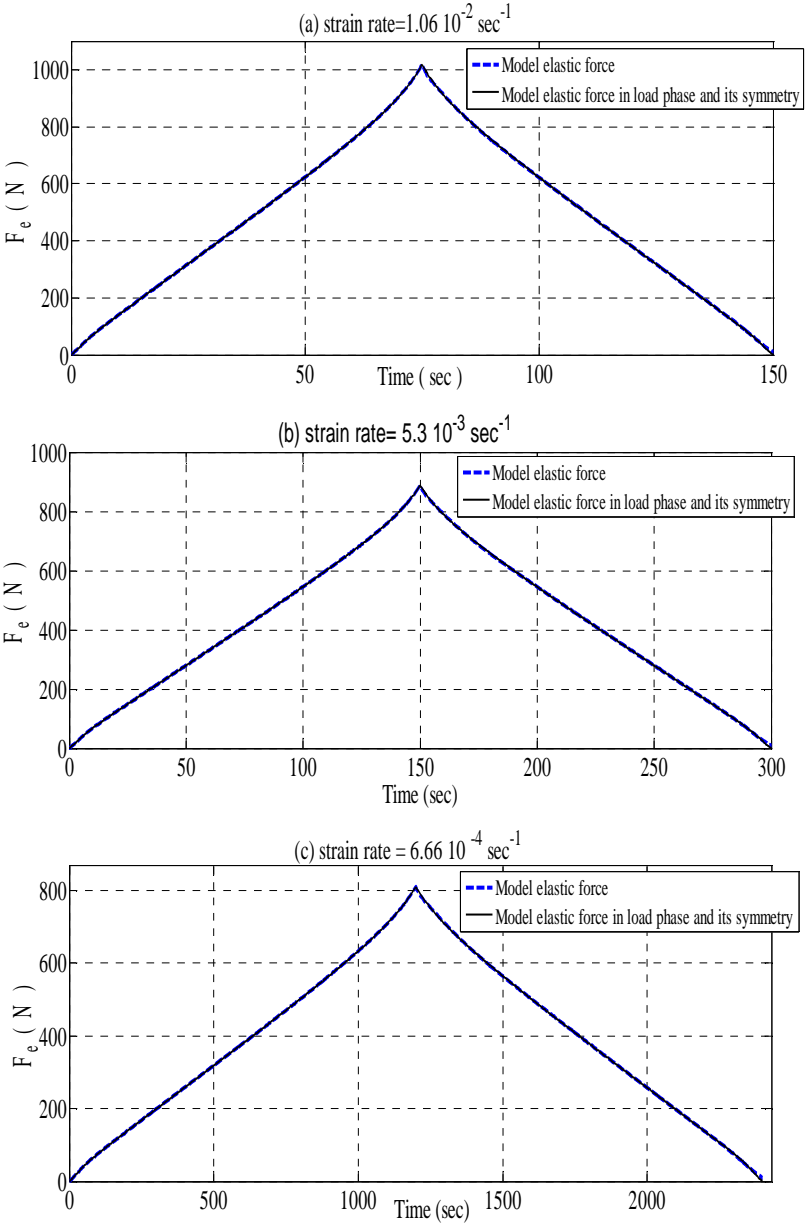
The comparison of model and experimental difference-forces between load and unload phases is shown in figure 5. The maximum relative error between experimental and analytical curves is in the order of 5%. It is also shown in figure 4 that the initial part of the analytical curve is decaled comparing with the experimental curve. The explication of this remark is determined through interpretation of figure 6. In this figure, it is shown that the model reconstructs the foam force response in good similarity with experimental force response. Only in the end of test, the model and experimental responses are not same and the

difference is very important. In fact, a residual stress obliges foam to return into a final position different to initial before test. This is a result of viscoelasticity behaviour. So the contact between upper frame and foam is lost and the experimental force response is inaccurate.





**Figure 6.** Comparison experimental force response and model force response.



**Figure 7.** Verification of elastic force symmetry.

The figure 6 shows that the foam model anticipates force response between minimum and maximum experimental envelopes.

The symmetry of the elastic force between load and unload phases is the basis assumption of difference-forces method. Thus, it is obligatory to verify symmetry condition to validate identification parameters. The reconstruction of global elastic response shows that the model results verify elastic symmetry (figure 7).

### 5.3. Discussion of results

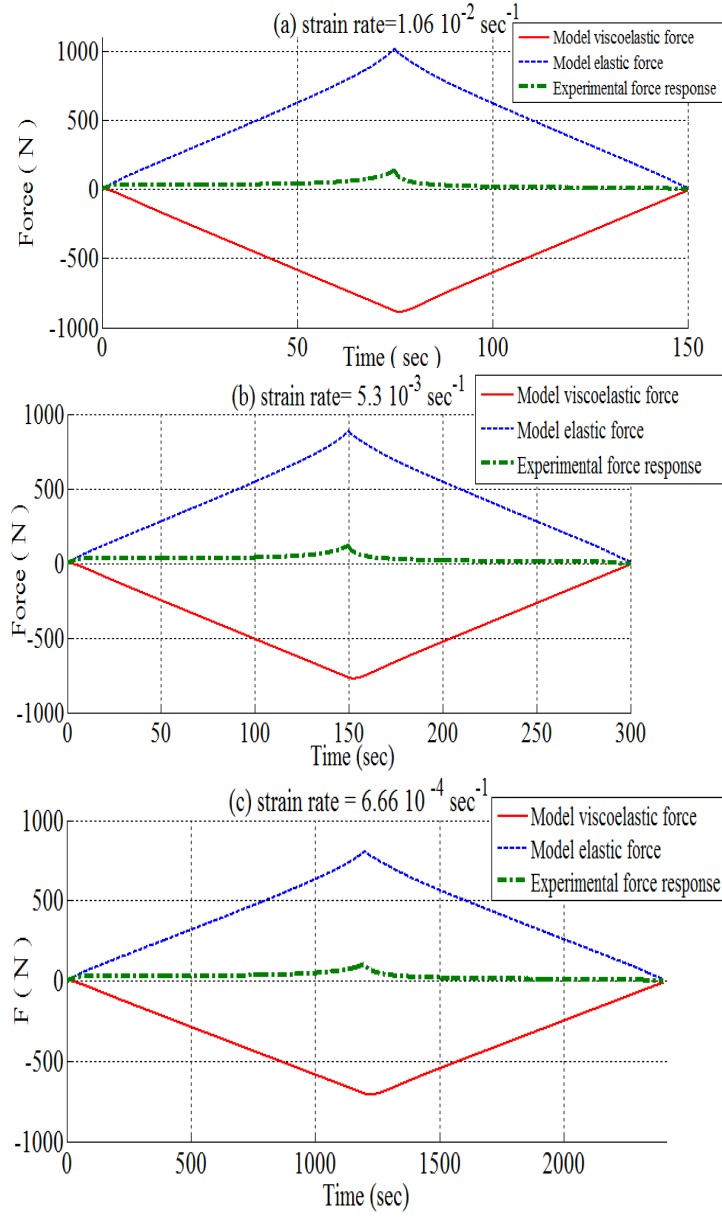
We have shown in this article that it is possible to characterize the viscoelastic behaviour of polyurethane foam. Unfortunately, these parameters are function of the test conditions such as strain rate. This is undoubtedly due to the foam complex compartment, that can't be modelled by a combination of viscoelastic models with constant parameters like Kelvin-Voigt or Maxwell models. Moreover our results aren't in contradiction with bibliography (Deng *et al.*, 2006). The introduction of dimensionless parameters, independent of test conditions could be a possible way to characterize the viscoelastic behaviour of polyurethane foam. Moreover, the use of dimensionless parameters could make algorithm faster with ameliorating conditioning matrix used in estimation method.

However, the order values of global elastic response and viscoelastic response (Figure 8) is very important. The global elastic polynomial is almost linear (figure 7). This result contradicts the works presented in the introduction and the firstly assumption: foam under large deformation exhibits nonlinear elastic behaviour. The linearity is due to the dominance of the global stiffness response of first order [13].

$$k_1 x \gg \sum_{i=2}^8 k_i x^i \quad [13]$$

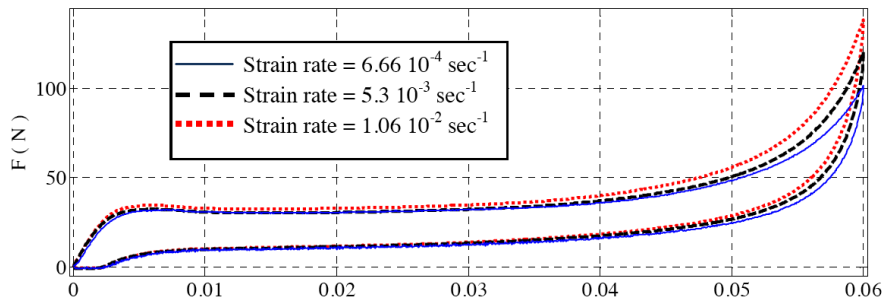
This stiffness  $k_I$  can be composed in two terms: spring elastic stiffness  $K_I$  and viscoelastic origin stiffness  $K_{ve}$  (equations 2, 5 and 7).

Note that, in this paper,  $M$  is considered to be equal to eight for future comparison with fractional model (Jmal *et al.*, 2011).



**Figure 8.** Reconstruction elastic and viscoelastic forces.

The comparison between the foam force response in various strain rates (Figure 9.) shows that the unload phases are similar for all strain rates. So, it seems possible to extract elastic information about it. This idea constitutes a good way for the further work to ameliorate the integer model.



**Figure 9.** Force response of foam in various strain rates.

## 6. Conclusion

The difference-method is based on elastic symmetry between load and unload phases. This method allows us to identify the viscoelastic and global elastic parameters of macroscopic integer model. The parameters resulting verify the thermodynamic conditions, the symmetry assumption, a good statistical validity and the insertion of the curve into the experimental corridor.

However, the integer model cannot describe the response of nonlinear spring. To solve this problem, we would be obliged to separate the first order stiffness in the part of nonlinear elastic compartment and the part of elasticity contained in the viscoelastic compartment.

Indeed, the introduction of dimensionless viscoelastic parameters, invariant with test conditions, is a good further approach to characterize the foam material.

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**8. Nomenclature**

Symbols	Units	Definitions
$t$	(sec)	Time
$T$	(sec)	Test period
$T_{ec}$	(sec)	Sampling period
$\omega$	(Hz)	Frequency
$x$	(m)	Displacement
$(L_0 \times l_0 \times h_0)$	(m $\times$ m $\times$ m)	Initial dimensions of polyurethane foam samples
$\varepsilon$	(m m <sup>-1</sup> )	Strain
$\dot{\varepsilon}$	(sec <sup>-1</sup> )	Strain rate
$\varepsilon_{\max}$	(m m <sup>-1</sup> )	Maximum strain
$\varepsilon_0$	(m m <sup>-1</sup> )	Initial strain
$F$	(N)	Foam force response
$F_{ve}$	(N)	Viscoelastic force
$F_e$	(N)	Elastic force (spring force)
$K_i$	(N m <sup>-1</sup> )	Spring stiffness of order $i$
$k_i$	(N m <sup>-1</sup> )	Global elastic stiffness of order $i$
$K_{ve}$	(N m <sup>-1</sup> )	Viscoelastic stiffness
$\alpha_l$	(sec <sup>-1</sup> )	Complex number representing $l^{th}$ viscoelastic mode
$a_l$	(N m <sup>-1</sup> sec <sup>-1</sup> )	Complex number design $l^{th}$ viscoelastic residue
$P$		Number of viscoelastic mode
$n$		Integer number representing the derivation order associate to displacement
$r$		Integer number representing the derivation order associate to viscoelastic
$(c_i)_{i=0..n}$		Real coefficients associated to displacement derivation of order $i$
$(b_i)_{i=0..r}$		Real coefficients associated to viscoelastic force derivation of order $i$
$M$		Order of elastic polynomial
$N$		Minimum number of test samples